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ASYMMETRIC EQUILIBRIA IN SPATIAL COMPETITION*

by

Takatoshi Tabuchi and Jacques–François Thisse

Kyoto University | Université de Paris I–Sorbonne and CERAS–ENPC

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ABSTRACT

The main purpose of this paper is to study the impact of consumer concentration around the market center on the equilibrium locations of location–price games. In the case of symmetric triangular density, it is shown that no symmetric equilibrium exists. However, we demonstrate the existence of asymmetric equilibria in pure strategies; these equilibria are also characterized. Our secondary purpose is to study the sequential entry of two firms when the location space is not restricted to the market space. This leads us to uncover a substantial first–mover advantage, which has been neglected in the literature.

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1. INTRODUCTION

Ever since the pioneering contribution of Hotelling (1929), most of the literature on spatial competition has assumed a uniform distribution of consumers (see Gabszewicz and Thisse (1992) for a recent survey and references). Clearly, such a simplifying assumption is due to its mathematical tractability. However, research in marketing has pointed out the existence of "consumer pockets" in the characteristics space, corresponding to customers whose preferences are clustered around some fashionable brands (see, e.g. Kuehn and Day (1962)). Similarly, in the urban setting, it is well known that the distribution of households is concentrated around the central business district (see, e.g. Clark (1951)). Therefore, the need to consider non–uniform distributions is apparent.

In this paper, we study the equilibrium locations of location–price games when consumers are concentrated around the market center. For this purpose, we consider the simple case of a symmetric and triangular density of consumers. The higher concentration of consumers around the center suggests that firms would move toward more central locations than in the case of a uniform density. This a priori reasonable conjecture is not confirmed by the analysis. Somewhat surprisingly, there exists no symmetric location equilibrium in this model. This is because the best reply functions are discontinuous when firms are symmetrically located. However, asymmetric equilibria turn out to exist. Thus, in equilibrium, one firm is strictly better–off than its rival despite the fact that the firms compete under identical conditions. As discussed in the concluding senction, this is not an artifact of triangular density. The same results hold for a wide class of convex densities such as the negative exponentials, thus casting some doubts on the robustness of results derived under the uniform density assumtion. Furthermore, we relax the standard assumption that firms must locate inside the market space. To our surprise, other equilibria emerge when firms are free to choose their locations outside the market space.

The secondary purpose of this paper is to revisit Hotelling's duopoly model in light of Prescott and Visscher's (1977) approach. These authors observe that many real world location decisions are not made simultaneously, but rather sequentially. Given that most location decisions are irrevocable, the first entrant will strive to anticipate the choices of the subsequent entrants. In this paper, we limit ourselves to the case of two firms. Both firms enter the market sequentially but choose their prices simultaneously. Our formulation of sequential entry differs, therefore, from that adopted by Anderson (1987) who considers a two–stage Stackelberg game in location and price. Since, in most cases, prices can be revised after the entry of a new firm, a simultaneous Nash equilibrium seems indeed to be more appropriate to model price competition. Moreover, here also, we do not assume that the location space is restricted to the market space. This leads to a substantial first–mover advantage, unlike Neven (1987) who assumes that firms locate inside the market space.

The remainder of the paper is organized as follows. The model is described in Section 2. In Section 3, we explore the existence of a subgame perfect Nash equilibria in pure strategies for a location–price game in which the consumer density is triangular. In Section 4, the assumption of simultaneous location choices is replaced with that of sequential choices and the corresponding equilibrium is analyzed. Section 5 concludes the paper.

2. THE MODEL

There are two firms producing a homogeneous good at a constant and equal marginal cost, which is set equal to zero. There is a continuum of consumers distributed over the unit segment $[0,1]$ and their location is denoted by $x \in [0,1]$. Let $F(x)$ be the cumulative distribution of consumers, where the total population $F(1)$ is normalized to one, and $f(x)$ be the corresponding density. Two distributions are considered in this paper: (i) the *uniform* density in which $f(x)=1$ for all $x \in [0,1]$; (ii) the *triangular* density in which $f(x)=2-2|2x-1|$ for all $x \in [0,1]$. We retain these two distributions because the uniform density is commonplace in the literature and because the triangular one is the simplest density that captures the idea of consumer concentration about one point. The transportation cost incurred by consumers is assumed to be a quadratic function of distance (without loss of generality, the transportation coefficient is normalized to one). Each consumer buys one unit of the good from the firm having the lower full price (i.e., mill price plus transportation cost).

When firms are set up at $x_1 \neq x_2$ in $\mathbb{R}$, the marginal consumer, who is indifferent between purchasing from either firm, is located at $\hat{x}$ as given by:

$$\hat{x} \equiv (p_2 - p_1 + x_2^2 - x_1^2)/2(x_2 - x_1). \tag{1}$$

When $x_1 < x_2$, the firms' profit functions are respectively

$$\Pi_1 = p_1 F(\hat{x}) \quad \text{and} \quad \Pi_2 = p_2[1 - F(\hat{x})]. \tag{2}$$

For $x_1 > x_2$ they are

$$\Pi_1 = p_1[1 - F(\hat{x})] \quad \text{and} \quad \Pi_2 = p_2 F(\hat{x}). \tag{3}$$

Finally, when $x_1 = x_2$, the profit functions are as in the Bertrand game.

We seek subgame perfect Nash equilibria. Hence, we solve the game by backward induction, starting from the last stage: given x_1 and x_2, firms choose simultaneously their (mill) price p_1 and p_2 with $p_1, p_2 \geq 0$. The following result, due to Caplin and Nalebuff (1991), guarantees the existence of a price equilibrium in pure strategies for a wide class of consumer density functions that includes the uniform and triangular ones.

Proposition 1

If the transportation cost is quadratic in distance, then for any given locations of firms and for any log-concave consumer density function,[1] *there exists a Nash price equilibrium. Furthermore, this equilibrium is unique.*

Assuming $x_1 < x_2$, the profit functions are differentiable, and the first-order conditions for equilibrium prices are given by

$$\frac{\partial \Pi_1}{\partial p_1} = F(\hat{x}) - \frac{p_1 f(\hat{x})}{2(x_2 - x_1)} = 0, \tag{4a}$$

$$\frac{\partial \Pi_2}{\partial p_2} = 1 - F(\hat{x}) - \frac{p_2 f(\hat{x})}{2(x_2 - x_1)} = 0. \tag{4b}$$

We know from Proposition 1 that (4a) and (4b) yield a unique price equilibrium when $f(x)$ is log-concave. Expressions similar to (4a) and (4b) can be obtained for $x_1 > x_2$.

Regarding the earlier stage(s) of the game, we assume in section 3 that firms choose *simultaneously* their locations x_1 and x_2 in $\mathbb{R}$, while in section 4 we suppose that firms select their locations *sequentially*. In both cases, they anticipate the outcome of the subsequent price stage given by Proposition 1.

3. SIMULTANEOUS LOCATION CHOICE

We assume here that both firms select their location simultaneously and then, after having observed the decisions made, choose their price simulaneously. When $x_1=x_2$, the profit functions are $\Pi_1^*(x_1,x_2)=\Pi_2^*(x_1,x_2)=0$. Suppose now that $x_1<x_2$ (without loss of generality, this assumption is made throughout this section and the next one). Plugging the first–order conditions (4a)–(4b) for equilibrium prices into (2) yields the payoff functions of the location game:

$$\Pi_1^*(x_1,x_2) = 2(x_2-x_1)F^2(\hat{x})/f(\hat{x}), \tag{5a}$$

$$\Pi_2^*(x_1,x_2) = 2(x_2-x_1)[1-F(\hat{x})]^2/f(\hat{x}). \tag{5b}$$

A Nash location equilibrium, $\underline{x}^N=(x_1^N,x_2^N)$ is such that firm i maximizes $\Pi_i^*(x_i,x_j^N)$ with respect to x_i (i,j=1,2 and i≠j). Clearly, the agglomeration of the two firms ($x_1=x_2$) is never an equilibrium of the location game since the profits are zero. Furthermore, for any given location pair, $\hat{x}$ can be determined by solving the equation

$$2F(\hat{x}) - 1 + (\hat{x}-x_1/2-x_2/2)f(\hat{x}) = 0, \tag{6}$$

which is obtained directly by subtracting (4b) from (4a) and by replacing p_2-p_1 in (1). At any location equilibrium, $\hat{x}\in]0,1[$ since otherwise one firm would be driven out of business. Furthermore, $\hat{x}=1/2$ if and only if firms have symmetric locations.

In the uniform density case, (6) shows that

$$\hat{x} = (2+x_1+x_2)/6. \tag{7}$$

With the triangular density, two cases arise. If $x_1+x_2<1$, then

$$\hat{x} = \frac{x_1+x_2+\sqrt{(x_1+x_2)^2+8}}{8}; \tag{8a}$$

if $x_1+x_2>1$, then

$$\hat{x} = \frac{x_1+x_2+6-\sqrt{(x_1+x_2-2)^2+8}}{8}. \tag{8b}$$

It is easy to check that (8a) and (8b) are equal to 1/2 when the two firms are located symmetrically ($x_1+x_2=1$). When $\hat{x}$ is differentiable, $\delta \equiv \partial\hat{x}/\partial x_1 = \partial\hat{x}/\partial x_2 > 0$.

If $f(\hat{x})$ is differentiable, we have

$$\frac{\partial \Pi_1^*}{\partial x_1} = -\frac{F^2(\hat{x})}{f(\hat{x})} + 2(x_2-x_1)F(\hat{x})\delta - (x_2-x_1)\frac{F^2(\hat{x})f'(\hat{x})\delta}{f^2(\hat{x})} = 0, \tag{9a}$$

$$\frac{\partial \Pi_2^*}{\partial x_2} = \frac{[1-F(\hat{x})]^2}{f(\hat{x})} - 2(x_2-x_1)[1-F(\hat{x})]\delta - (x_2-x_1)\frac{[1-F(\hat{x})]^2 f'(\hat{x})\delta}{f^2(\hat{x})} = 0. \tag{9b}$$

Dividing (9a) by $F(\hat{x})$ and (9b) by $1-F(\hat{x})$, adding these two expressions, and substituting $(x_2-x_1)\delta$ from (9b), we obtain after simplifications

$$H(\hat{x}) \equiv [1-2F(\hat{x})]f^2(\hat{x}) - [1-F(\hat{x})]F(\hat{x})f'(\hat{x}) = 0. \tag{10}$$

This equation must be satisfied for any location equilibrium such that $f'(\hat{x})$ exists.

3.1 Consider first the case of a uniform density. Clearly, $\hat{x}=1/2$ is the only solution to (10) so that the equilibrium locations must be symmetric. Using (7), (5a) and (5b) can be rewritten as

$$\Pi_1^*(x_1,x_2) = (x_2-x_1)(2+x_1+x_2)^2/18, \tag{11a}$$

$$\Pi_2^*(x_1,x_2) = (x_2-x_1)(4-x_1-x_2)^2/18. \tag{11b}$$

Because the strategy space of firm 1 is unbounded and because its payoff is continuously differentiable everywhere, firm 1's equilibrium location must satisfy the first–order condition as an equality. Differentiating (11a) with respect to x_1 yields after simplifications $-2-3x_1+x_2=0$. Since we may restrict ourselves to a symmetric solution ($x_1+x_2=1$), the candidate equilibrium locations are given by

$$x_1^N = -1/4 \quad \text{and} \quad x_2^N = 5/4. \tag{12}$$

The second–order conditions of maximization of Π_1^* and Π_2^* being satisfied at these values, (12) is the unique Nash equilibrium of the location game. Hence, *under a uniform distribution, firms choose to locate outside the market.* This unexpected result reflects the fact that price competition under quadratic transportation costs is very fierce indeed. However, firms do not want to set up at infinity because the two best reply curves are linear and not parallel.

3.2 We now move to the triangular density. Our first result rules out the possibility of symmetric equilibria.

Proposition 2

For the symmetric triangular distribution of consumers, there exits no symmetric location equilibrium.

Proof: Since the distribution is triangular, we can compute Π_1^* defined by (5a) when x_1 and x_2 are almost symmetric about the market center. After some standard, but tedious, calculations, we obtain

$$\Pi_1^*(1-x_2-\epsilon,x_2) - \Pi_1^*(1-x_2,x_2) = (1-x_2)\epsilon/2 + O(\epsilon^2),$$

and

$$\Pi_1^*(1-x_2+\epsilon,x_2) - \Pi_1^*(1-x_2,x_2) = 5(x_2-4/5)\epsilon/12 + O(\epsilon^2),$$

where $\epsilon>0$ is small enough. That is: (a) for $x_2 \leq 4/5$, we have $\Pi_1^*(1-x_2-\epsilon,x_2) > \Pi_1^*(1-x_2,x_2)$; (b) for $4/5<x_2<1$, $\Pi_1^*(1-x_2-\epsilon,x_2)>\Pi_1^*(1-x_2,x_2)$ and $\Pi_1^*(1-x_2+\epsilon,x_2)>\Pi_1^*(1-x_2,x_2)$; (c) for $x_2 \geq 1$, $\Pi_1^*(1-x_2+\epsilon,x_2)>\Pi_1^*(1-x_2,x_2)$. Therefore, when x_1 and x_2 are symmetric about the center, x_1 is not firm 1's best reply against x_2. The same holds for firm 2. ■

The nonexistence of a symmetric equilibrium is somewhat surprising since our game involves two identical firms competing under identical conditions. It is due to the discontinuity of the best reply function when $x_1+x_2=1$ (see Figure 1 for an illustration). This discontinuity itself arises because of the discontinuity of f' at the market center. When firms are symmetrically located but far enough, the discontinuity of $f'(\hat{x})$ at $\hat{x}=1/2$ makes an infinitesimal move inward profitable because the firm gains a whole strip of consumers. On the other hand, when firms are symmetrically located but not far from each other, the discontinuity makes an infinitesimal move outward profitable because prices steeply increase (as shown by differentiating (4a) with respect to x_1). In a more formal way, we observe that the profit function Π_1^* (Π_2^*) is not quasi–concave: when $4/5<x_2<1$ (see case (b) in the above proof), the symmetric configuration corresponds to a local minimum of $\Pi_1^*(\Pi_2^*)$.

[Insert Figure 1 about here]

However, asymmetric equilibria may exist as shown below.

Proposition 3

When the distribution of consumers is triangular, there exist two asymmetric Nash location equilibria, which are given by

$$(x_1^N,x_2^N) = \begin{cases} (-\sqrt{6}/9, 5\sqrt{6}/18) \\ (1-5\sqrt{6}/18, 1+\sqrt{6}/9). \end{cases}$$

Proof: By Proposition 2, we may restrict ourselves to asymmetric location pairs. Since $\hat{x}\neq1/2$, $f'(\hat{x})$ exists. Solving (10), we get the following values for $\hat{x}$: $0, 1/\sqrt{6}, 1-1/\sqrt{6}, 1$. We have already seen that $\hat{x}=0$ and $\hat{x}=1$ must be ruled out. Hence, only two solutions are left: $\hat{x}=1/\sqrt{6}$ and $\hat{x}==1-1/\sqrt{6}$. The corresponding candidate equilibrium locations can be obtained from (6) and (9a): $(-\sqrt{6}/9, 5\sqrt{6}/18)$ and $(1-5\sqrt{6}/18, 1+\sqrt{6}/9)$. We show below that the first location pair is an equilibrium. Since the setting is symmetric about 1/2, this implies that the second pair is also an equilibrium.

(i) Let us show that $x_1^N=-\sqrt{6}/9$ is the best reply against $x_2^N=5\sqrt{6}/18$.

Replacing x_2 by x_2^N in (5a) and differentiating with respect to x_1 yields

$$\frac{\partial\Pi_1^*}{\partial x_1} = \frac{4\hat{x}^3}{8\hat{x}^2+1}[-16\hat{x}^2+(10/\sqrt{6})\hat{x}+1],$$

when $\hat{x}\in]0,1/2[$. The sign of $\partial\Pi_1^*/\partial x_1$ is given by the sign of $G_1(\hat{x})\equiv -16\hat{x}^2+(10/\sqrt{6})\hat{x}+1$. Clearly, $G_1(\hat{x})\gtreqless0$ for $\hat{x}\lesseqgtr1/\sqrt{6}$, which corresponds to $x_1^N\lesseqgtr-\sqrt{6}/9$. Therefore, in order to prove that $x_1^N=-\sqrt{6}/9$ yields a global maximum, it suffices to show that $\partial\Pi_1^*/\partial x_1\leq0$ for all

$\hat{x}\in]1/2,1[$ because Π_1^* is continuous with respect to $\hat{x}$.

When $\hat{x}\in]1/2,1[$, some tedious calculations show that

$$\frac{\partial\Pi_1^*}{\partial x_1}=\frac{1-2(1-\hat{x})^2}{(1-\hat{x})[8(1-\hat{x})^2+1]}[32(1-\hat{x})^4-\frac{36-10\sqrt{6}}{3}(1-\hat{x})^3-2(1-\hat{x})^2-\frac{18-5\sqrt{6}}{9}(1-\hat{x})-1].$$

The sign of $\partial\Pi_1^*/\partial x_1$ is given by the sign of $G_2(\hat{x})$ defined by the bracketed term at the numerator of this expression. It is readily verified that $G_2(1/2)$ and $G_2(1)$ are both negative. Furthermore, studying the first and the second order derivatives of $G_2(\hat{x})$ shows that this function first decreases and then increases over the interval $]1/2,1[$. Hence, $G_2(\hat{x})<0$ for all $\hat{x}\in]1/2,1[$ so that $x_1^N=-\sqrt{6}/9$ is firm 1's best reply against $x_2^N=5\sqrt{6}/18$.

(ii) We next prove that $x_2^N=5\sqrt{6}/18$ is firm 2's best reply against $x_1^N=-\sqrt{6}/9$. The argument is similar to the one above. First, for $\hat{x}\in]0,1/2[$, we have

$$\frac{\partial\Pi_2^*}{\partial x_2}=\frac{\hat{x}-1/\sqrt{6}}{8\hat{x}^3+\hat{x}}[-64\hat{x}^3(17/48-\hat{x}^2)-40\sqrt{6}\hat{x}^2(7/20-\hat{x}^2)/3-14\hat{x}/3-\sqrt{6}].$$

Since the bracketed term is negative, it is clear that $\partial\Pi_2^*/\partial x_2\gtreqless 0$ for $\hat{x}\lesseqgtr 1/\sqrt{6}$, which corresponds to $x_2\lesseqgtr 5\sqrt{6}/18$. Therefore, in order to prove that $x_2^N=5\sqrt{6}/18$ yields a global maximum, it suffices to show that $\partial\Pi_2^*/\partial x_2\leq 0$ for all $\hat{x}\in]1/2,1[$.

When $\hat{x}\in]1/2,1[$, we have

$$\frac{\partial\Pi_2^*}{\partial x_2}=-\frac{2(1-\hat{x})^3}{8(1-\hat{x})^2+1}[(2\hat{x}-1)(9-8\hat{x})+4(1-\hat{x})/\sqrt{6}],$$

which is negative for all $\hat{x}\in]1/2,1[$. ∎

The market share of firm 1 is $F(1/\sqrt{6})=1/3$ at the first location equilibrium, and $F(1-1/\sqrt{6})=2/3$ at the second one. The corresponding profits are (7/54,14/27) and (14/27,7/54) respectively. These profit values are less than the profit earned by each firm in the uniform case. The concentration of consumers around the center attracts the two firms, which reduces the distance between them and intensifies price competition. The result is a decrease in the equilibrium profits of both firms.[3]

Unlike the uniform distribution, the triangular distribution, though symmetric, leads to asymmetric locations, prices and profits. In other words, *competition between two identical firms results in asymmetric locations* if consumers are symmetrically concentrated around the center. This shows the lack of robustness of the symmetric equilibrium which often appears in the literature on spatial competition. However, the existence of two asymmetric equilibria, in which firms make different profits, leaves open the question of which equilibrium arises.

4. SEQUENTIAL LOCATION CHOICE

Until now, we have focused on the simultaneous game in location. However, as discussed in the introduction, it may be more realistic to assume that firms enter the market *sequentially* while price competition remains simultaneous. More precisely, there are now three stages. The first two stages describe a Stackelberg game in location while the third stage is a simultaneous subgame in price.

Formally, the model of Section 2 has to be modified in the following manner. Firm 1 (the leader) maximizes its profit $\Pi_1^*(x_1,x_2)$ with respect to x_1, replacing x_2 by firm 2 (the follower)'s best reply function $x_2=R(x_1)$, which is itself derived from the maximization of $\Pi_2^*(x_1,x_2)$ with respect to x_2. The resulting Stackelberg location equilibrium is denoted by $\underline{x}^S=(x_1^S,x_2^S)$. Assuming that $R(x_1)$ is single–valued and differentiable, the first–order conditions for such an equilibrium are as follows:

$$\frac{d\Pi_1^*}{dx_1}=\frac{\partial\Pi_1^*}{\partial x_1}+\frac{\partial\Pi_1^*}{\partial x_2}\frac{dR}{dx_1}, \tag{13a}$$

$$\frac{\partial\Pi_2^*}{\partial x_2}=0, \tag{13b}$$

where dR/dx_1 is equal to $-(\partial^2\Pi_2^*/\partial x_2\partial x_1)/(\partial^2\Pi_2^*/\partial x_2^2)$. Solving (6) and (9b) with respect to x_1 and x_2, and replacing these variables in (13a), we get:

$$\frac{d\Pi_1^*}{dx_1}=\frac{4F(\hat{x})K(\hat{x};x_1)}{\{2f^2(\hat{x})+[1-F(\hat{x})]f'(\hat{x})\}f(\hat{x})},$$

where

$$K(\hat{x};x_1)\equiv[1-2F(\hat{x})+R'(x_1)]f^2(\hat{x})-[1-F(\hat{x})]F(\hat{x})f'(\hat{x}). \tag{14}$$

Using (11b), the denominator of (14) can be shown to be positive for all $\hat{x}\in]0,1[$ so that $\text{sgn}(d\Pi_1^*/dx_1)= \text{sgn}(K(\hat{x}))$. This property is used in the proposition below.

Proposition 4

If the distribution of consumers is uniform or triangular, then the first entrant necessarily locates at the market center.

Proof:

(i) Uniform distribution

Some standard manipulations show that firm 2's best reply function is $R(x_1)=x_1/3+4/3$. Using this expression and (7), (14) shows that $\text{sgn}(d\Pi_1^*/dx_1)=\text{sgn}(1-2x_1)$. Therefore, the optimum location of the first entrant is $x_1^S=1/2$.

(ii) Triangular distribution

Because f(x) is symmetric, it is sufficient to show that $d\Pi_1^*/dx_1>0$ for all $x_1\in]-\infty,1/2[$ such that $\hat{x}\in]0,1[$.

(a) Assume first that $\hat{x}\leq 1/2$.

By solving (6) and (9b) with respect to x_1 and x_2, it can be shown that (5a) depends only upon $X\equiv\hat{x}^2\in[0,1/4]$:

$$\Pi_1^*(X) = \frac{X(-32X^3+28X^2-4X-1)}{12X^2-4X-1}.$$

Differentiating this expression with respect to X, we get

$$d\Pi_1^*(X)/dX = \frac{L(X)}{(12X^2-4X-1)^2},$$

where $L(X) \equiv -768X^5+720X^4-96X^3-56X^2+8X+1$. Differentiating L(X) yields $L'(X)=8(1-2X)M(X)$, where $M(X)\equiv 1-12X-60X^2+240X^3$. Differentiating M(X) gives $M'(X)=12(-1-10X+60X^2)$. Since $M'(0)<0$ and $M'(1/4)>0$, M'(X) changes its sign only once in the interval [0,1/4]. Moreover, since $M(0)>0$ and $M(1/4)<0$, M(X), and hence L'(X), changes its sign only once in [0,1/4]. Consequently, $d\Pi_1^*(X)/dX$ is first increasing and then decreasing on [0,1/4] because $\mathrm{sgn}(d\Pi_1^*(X)/dX)=\mathrm{sgn}(L(X))$. Since $d\Pi_1^*/dX>0$ at $X=0$ and $X=1/4$, and since $\partial X/\partial x_1>0$, we can conclude that $\mathrm{sgn}(d\Pi_1^*/dx_1)=\mathrm{sgn}(d\Pi_1^*(X)/dX)$ is positive for all $X\epsilon[0,1/4]$, i.e., for all $\hat{x}\epsilon[0,1/2]$.

(b) Suppose now that $\hat{x}>1/2$.

Computing $K(\hat{x};x_1)$ as given by (14), we get

$$\mathrm{sgn}(d\Pi_1^*/dx_1) = \mathrm{sgn}(6(1-\hat{x})^2-1+2R'(x_1)), \text{ for } \hat{x}\epsilon]1/2,1]. \quad (15)$$

It therefore remains to show that (15) is positive.

Setting (9b) equal to zero and using (8b), we get firm 2's best reply function $R(x_1)$. Differentiating $R(x_1)$ with respect to x_1, we obtain

$$R'(x_1) = -\frac{1}{4} + \frac{9(1-x_1)}{4C} + \frac{24[1+3(1-x_1)/C]}{(3-3x_1+C)^2}$$
$$> -\frac{1}{4} + \frac{9(1-x_1)}{20} + \frac{24}{5(8-3x_1)} \equiv N(x_1),$$

where $C \equiv \sqrt{9(1-x_1)^2+16} < 5$ for all $x_1\epsilon]0,1/2]$. Indeed, since $\hat{x}>1/2$, $x_2=R(x_1)$ lies above $x_1+x_2=1$ which implies that x_1 must be positive as shown by Figure 1.

However, since $N'(x_1) = 9(-9x_1^2+48x_1-32)/[20(3x_1-8)^2]<0$ on [0,1/2], we have $R'(x_1)>N(1/2)=371/520$. Replacing $R'(x_1)$ by this value in (15), we see that $\mathrm{sgn}(d\Pi_1^*/dx_1)= \mathrm{sgn}(6(1-\hat{x})^2+111/260)$, which is positive for all $\hat{x}\epsilon]1/2,1]$. ■

The Stackelberg equilibrium locations are then summarized as follows:

(i) for the uniform distribution,

$$(x_1^S,x_2^S) = \begin{cases} (1/2,\ 3/2) \\ (1/2,-1/2) \end{cases}$$

and

$$(\Pi_1^*(\underline{x}^S),\Pi_2^*(\underline{x}^S)) = (8/9,2/9);$$

(ii) for the triangular distribution,

$$(x_1^S,x_2^S) = \begin{cases} (1/2,\ 1.443) \\ (1/2,-0.443) \end{cases}$$

and

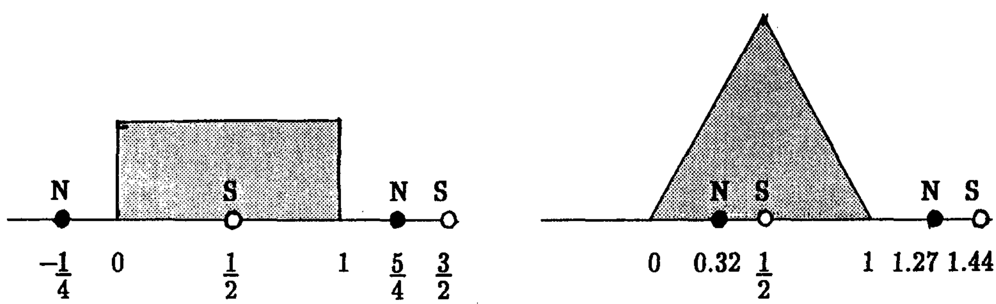
$$(\Pi_1^*(\underline{x}^S),\Pi_2^*(\underline{x}^S)) = (0.715,0.089).$$

In either case, *the first–mover advantage is substantial.* The profit of the firm 1 is four times as large as that of firm 2 in the uniform case, and approximately eight times in the triangular case. The latter exhibits a larger profit differential because of the higher concentration of consumers around the center where the first entrant locates.

Furthermore, under the uniform distribution of consumers, we have $(x_1^S,x_2^S)=(0,1)$ when the location space is restricted to [0,1]. When this constraint – the justification of which is far from being obvious to us – is relaxed, we obtain a totally different pattern as one firm (the leader) locates at the market center and the other (the follower) outside the market space.

5. CONCLUSIONS

Our main results can be illustrated by the following two diagrams:



where N stands for the simultaneously chosen locations and S for the sequential ones.

The following remarks are in order.

(i) It can be shown that Proposition 2 still holds when the triangular density has positive and equal values at the market endpoints, however close it is to the uniform density. The result also remains valid for convex but log–concave densities, such as the negative exponentials, which are very popular in urban population distribution models (see Proposition 3 in Tabuchi and Thisse (1989)). Thus, moving from the uniform density may destroy the existence of a symmetric equilibrium, even though the model remains symmetric. This suggests that the systematic emphasis put on symmetric equilibria in standard models of spatial competition is not well founded, and invites us to pay more attention to asymmetric equilibria.

(ii) When the density is concave, symmetric and not "too much" different from the uniform density, there exists a unique Nash location equilibrium, which is symmetric (see Proposition 2 in Tabuchi and Thisse (1989)). More precisely, while firms locate at the outside quartiles when the density is uniform, they locate closer to the market center as the density becomes more concentrated. Eventually, they will lie inside the market. In such cases, the price competition effect is outweighed by the demand effect generated by the high concentration of consumers around the center. According to Neven (1986, Proposition 3), the distance to the center from any equilibrium location is greater than 3/8. In view of all those results, it appears that *the set of location equilibria is very sensitive to the specification of the consumer distribution,* thus making the derivation of general results very problematic.

(iii) As a final remark, let us say that the above analysis has also shed some light on the role of the assumption that firms must locate inside the market space. Indeed, relaxing this apparently innocuous assumption may lead to quite different equilibrium outcomes. For example, in the sequential location game, this yields a completely different locational configuration and uncovers a first–mover advantage which does not appear in the standard setting. Here also, these results invite us to revisit the models of horizontal product differentiation when the location space is unbounded.

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FOOTNOTES

1 A function is said to be log–concave when the logarithm of this function is concave. For example, the positive (negative) exponentials and the power functions are log–concave.

2 When the strategy space of firm location is restricted to [0,1], Proposition 3 is modified as follows: the equilibrium locations are $(0, (\sqrt{33}-3)/\sqrt{2\sqrt{33}+2})$ and $(1-(\sqrt{33}-3)/\sqrt{2\sqrt{33}+2}, 1)$ respectively while they are given by (0,1) in the uniform case.

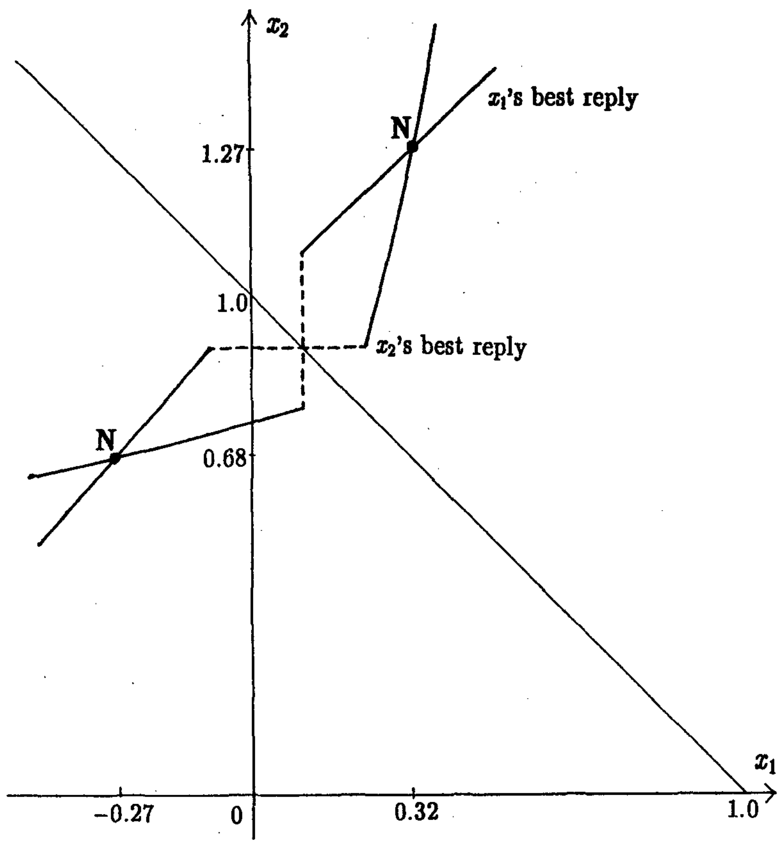

Figure 1 Best reply functions under the triangular consumer distribution